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The simple scheme for the calculation of the anomalous dimensions of composite operators in the $1/N$ expansion.

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Abstract

The simple method for the calculating of the anomalous dimensions of the composite operators up to $1/N^2$ order is developed. We demonstrate the effectiveness of this approach by computing the critical exponents of the $(\otimes\vec{\Phi})^s$ and $(\vec{\Phi}\otimes(\otimes\vec{\partial})^n\vec{\Phi})$ operators in the $1/N^2$ order in the nonlinear sigma model. The special simplifications due to the conformal invariance of the model are discussed.

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1 Introduction

The large N expansion provides the powerful tool for the investigating of the fields theoretic models with a internal symmetry. Being nonperturbative in its nature this method has proved to be successful in revealing properties which are inaccessible in conventional perturbation theory. The $1/N$ approach relies on the expansion of the effective action in the saddle point approximation which determines the leading order structure in $1/N$. However, beyond the lowest order the computations become quite untractable because the propagators of auxiliary fields depend on the mass in a very complicated fashion.

To obviate these difficulties it had been suggested in Refs. [1], where the nonlinear sigma model had been analyzed, to consider theory directly at critical point, that should ensure the masslessness of all propagators. The latter circumstance simplifies considerably the computation of the Feynman diagrams. Solving the skeleton Dyson equation the authors of Refs. [1, 2] have calculated the basic indices of sigma model with $1/N^2$ accuracy in the arbitrary space dimensions. Whilst the initial application of self consistency method was to a bosonic model, the techniques have also been extended to examine the fermionic theories [3]. The further progress in the calculation of the critical indices is related to the conformal bootstrap method. In the frame of this approach the most strong at the present moment results — $1/N^3$ anomalous dimensions of the basic fields in the σ model [4] and in the Gross -Neveu model [5, 6] — had been obtained.

If one is interested in the calculation of the critical indices related to the basic fields only, these methods – self - consistency equations and the conformal bootstrap method – are likely to be the most efficient. However, some specific features of the above methods make them inconvenient for the treatment of the composite operators, except for the simplest ones [7, 8]. At the same time, the calculations of this type became actual now. First of all, it is related with the attempts to understand in more details the structure of the conformal fields theories (CFT) in $D > 2$ dimensions (see Refs. [9, 10, 11] and references therein). The basic examples of higher dimensional CFT being bosonic and fermionic N vector models, the $1/N$ expansion underlies the computational method used for analyzing of these models. Second, the renewing of the interest to the $1/N$ calculations is conditioned by the recent progress in the higher order computations of the renormalization group (**RG**) functions in QCD. The analytical calculations in QCD in the high orders (four loop β - function [12], anomalous dimensions of twist - 2 operators [13, 14]) are very complex ones because of the huge number of Feynman diagrams to be evaluated. Thus it is important to have methods which may ensure the independent checks of this results. The one of the possible approach of this type is the $1/N_f$ expansion [15]. Further, the necessity of the careful analysis of the anomalous dimensions of the composite operators of special type have arisen in connection with the so - called stability problem [16, 17, 18]. Recently, it was realized that the $1/N$ expansion is the more suitable approach for this purpose [19].

As mentioned above, the calculations in $1/N$ scheme beyond the $1/N$ order become tractable only when theory is considered directly at the critical point. But in this case the corresponding model lost the property of the multiplicative renormalizability and cannot be worked out with the help of the standard **RG** methods. It forces one to use other approaches applicable in this case. The methods of self - consistency equations and conformal bootstrap, discussed above, are applicable for the operators of the special type only. The approach based on the operator product expansion used in Refs [11, 10] is too involved to have a practical use in higher order calculations and limited by the models possessing of the conformal symmetry. The most convenient approach for the calculating of the anomalous dimensions of the composite operators in the first order of $1/N$ expansion had been developed in Ref [20]. The authors of Ref. [20] have exploited the property of the scale invariance of the correlators at critical point to obtain the anomalous dimensions

of arbitrary composite operators. The main result of the paper [20] is the formula expressing $1/N$ order anomalous dimensions via the Δ pole residues of the corresponding diagrams (Δ is the regularization parameter, analogous ϵ in dimensional regularization). Certainly, this way is the most effective in the case of the operators of arbitrary type.

At the straightforward generalization beyond the $1/N$ order (see Refs. [21, 19]) the above method lost many its attractive features. In the present paper we shall take advantage of the other approach [22] which allow to derive the simple formula for the anomalous dimensions in the $1/N^2$ order. To avoid the inconveniences connected with the absence of the multiplicative renormalizability we consider extended model which is reduced to the initial one by tuning of some parameters. The extended model can be analyzed by the standard renormalization group (**RG**) methods. To obtain the anomalous dimensions in the initial model we relate it with the corresponding **RG** - functions of the extended one. In the following we restrict ourselves to the nonlinear sigma model, the generalization on the other model being straightforward.

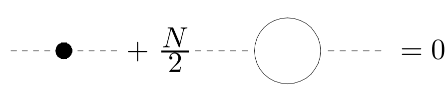
The paper is organized as follows: In section 2 we review the basic features of the nonlinear sigma model and discuss the problem to be solved. In section 3 the basic formulae for the anomalous dimensions of composite operators are derived. Some technical tricks useful in higher order calculations are collected in section 4. The results of $1/N^2$ order calculations of anomalous dimensions of some composite operators are presented in section 5. The conclusions are given in section 6.

2 Preliminaries

The $1/N$ - expansion for the massless $O(N)$ –nonlinear sigma model in $D \equiv 2\mu$-dimensions is derived in the standard manner from the following action:

$$S = -\frac{1}{2}(\partial\phi^A)^2 - \frac{1}{2}M^{-2\Delta}\psi K_\Delta \psi + \frac{1}{2}\psi(\phi^A)^2 + \frac{1}{2}\psi K \psi, \tag{2.1}$$

where ϕ^A is the $O(N)$ vector field and ψ–auxiliary field; Δ - regularization parameter, M renormalization mass. The two first term in (2.1) form the free part of the action, whereas two last are treated as a interaction. The kernel K is determined from the requirement of the cancellation of the simple ϕ loop insertion in ψ line



and reads as:

$$K(p) = -\frac{N}{2}\int \frac{\mathrm{d}^D k}{(2\pi)^D}\frac{1}{k^2(p-k)^2} = -\frac{N}{2(4\pi)^\mu}\frac{H^2(1)}{H(2\mu-2)}p^{D-4} \equiv b^{-1}p^{D-4}, \tag{2.2}$$

where $H(x) \equiv \Gamma(\mu - x)/\Gamma(x)$.

The regularized kernel K_Δ is defined as $K_\Delta = C^{-1}(\Delta)K(p)p^{2\Delta}$. The function $C(\Delta)$ is assumed to be regular at $\Delta = 0$ and $C(\Delta) \to 1$ at $\Delta \to 0$. The freedom in the choice of the form of $C(\Delta)$ will be discussed in Sec.3. The propagators of ϕ and ψ fields have form $(G_\psi = K_\Delta^{-1})$:

$$G_\phi^{AB}(p) = \delta^{AB}G_\phi(p) = \delta^{AB}p^{-2} \quad \text{and} \quad G_\psi(p) = bM^{2\Delta}C(\Delta)p^{-2(\mu-2+\Delta)} \tag{2.3}$$

in the momentum space and

$$G_\phi(x) = Ax^{-2\alpha} \quad \text{and} \quad G_\psi(x) = BM^{2\Delta}C(\Delta)x^{-2(\beta-\Delta)} \tag{2.4}$$

in the coordinate space. Here $\alpha = \mu - 1$, $\beta = 2$ and the explicit expressions for the amplitudes A, B can be easily deduced from the Eq. (2.3).

The renormalization of this theory carried out in the standard way. The divergences appears as Δ - pole terms in diagrams and can be excluded by the standard $\mathcal{R}$ - operation. The power counting shows that the theory with action (2.1) is renormalizable in the $1/N$ - expansion in the whole range $2 < D < 4$ and the renormalized action has the form:

$$S_R = -\frac{Z_1}{2}(\partial\phi)^2 - \frac{1}{2}M^{-2\Delta}\psi K_\Delta \psi + \frac{Z_2}{2}\psi\phi^2 + \frac{1}{2}\psi K \psi. \tag{2.5}$$

As distinct from the more usual field theoretic models, the theory under consideration is not multiplicatively renormalizable. Indeed, the renormalized action (2.5) cannot been brought into the form of that (2.1) by a redefinition of the fields (There is not a parameter like the coupling constant in (2.1)). Due to nonmultiplicativity, one fails to apply **RG** methods for the calculation of critical exponents and has to seek for another approaches. One of them, which is briefly sketched below relies on the property of the scale invariance of the theory. The latter can be deduced from the equivalence of the nonlinear sigma model to the $(\vec{\Phi}^2)^2$ theory in $D = 4 - \epsilon$ dimensions at the critical point. The more formal proof not appealing to such equivalence can be found in Ref. [22]. Now we shall discuss the basic points of the approach mentioned above. (For more details see Refs.[20, 21, 19].)

Let us consider the 1-irreducible Green function $\Gamma(p)$ that depends on the one momentum p only — the inverse propagator of ϕ field or the vertex function with one zero momentum. We prefer to work with the dimensionless object: $\bar{\Gamma}(p) = p^{-d}\Gamma(p)$, d being the canonical dimension of the function $\Gamma(p)$. Due to the scale invariance, the form of the renormalized Green function is fully determined (up to some constant factor $\bar{\Gamma}$) by its anomalous dimension γ:

$$\bar{\Gamma}(p) = \bar{\Gamma} \cdot \left(\frac{p}{M}\right)^{\gamma}. \tag{2.6}$$

The differentiation of the Eq.(2.6) with respect M yields the equation on the anomalous dimension

$$M\partial_M \bar{\Gamma}(p) = -\gamma\bar{\Gamma}(p). \tag{2.7}$$

To calculate the left side of the Eq.(2.7) we notice that the all dependence from M in the diagrams contributing to $\bar{\Gamma}(p)$ results from the propagators of ψ field (see Eq.(2.3)), i.e. the every diagram acquires the factor $M^{2\Delta n}$, where n is the number of ψ-lines in a diagram. Hence, the formula (2.7) takes the form:

$$\gamma\bar{\Gamma}(p) = -2\Delta \sum_{\{\Gamma_i\}} n_i \Gamma_i, \tag{2.8}$$

where n_i is the number of ψ-lines in diagram $\bar{\Gamma}_i$. The sum runs over all diagrams Γ_i for the renormalized Green function $\bar{\Gamma}(p)$ and, of course, the limit $\Delta \to 0$ is implied in Eq. (2.8).

In the first order of $1/N$ expansion Eq. (2.8) becomes very simple. Indeed, taking into account that the first order diagrams can develop the simple Δ poles only, $\Gamma_i = (\Gamma_i)_1/\Delta + (\Gamma_i)_0 + ...$, and that $\bar{\Gamma} = 1 + O(1/N)$ one gets

$$\gamma = -2 \sum_{\{\Gamma_i\}} n_i (\Gamma_i)_1 + O(1/N^2). \tag{2.9}$$

Thus for the calculation of the anomalous dimensions in the lowest order one need know the pole parts of the corresponding diagrams only. At first sight this property is lost in the next

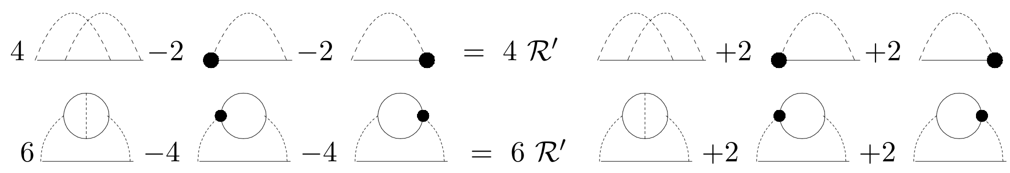


Figure 1: The example of the rearrangement of the diagrams. The black dots denote the vertex counterterm. The symbol $\mathcal{R}'$ stands for the standard operation of the subtracting of the subdivergencies from a diagram.

order. Indeed, the evaluating of the γ with $1/N^2$ accuracy requires the knowledge of the $1/N$ order corrections to the constant $\bar{\Gamma}$, which are determined by the finite parts of the $1/N$ order diagrams. This circumstance greatly reduces the effectiveness of the Eq. (2.8) for the high order calculations. To avoid this difficulty one may try to pick out of the sum in the Eq. (2.8) the diagrams reproducing the undesirable terms on the lhs of the latter. Taking in mind the analogous formulae in the dimensional regularization scheme we regroup the terms in the sum (2.8) in the way shown on the Fig 1. It can be checked, in these particular cases, that only the $\mathcal{R}'$ – terms contribute to the anomalous dimensions, whereas the others ensure the cancellation of the extra terms on the lhs of the Eq. (2.8). (The $\mathcal{R}'$ is the standard operation of the subtracting of the subdivergencies from diagram.) Eventually, the formula for the anomalous dimensions reads

$$\gamma = -2 \sum_{\{\Gamma_i\}} n_i (\mathcal{R}'\Gamma_i)_1 + O(1/N^3). \tag{2.10}$$

Here the sum runs over all diagrams (up to $1/N^2$ order) generated from the action (2.1), i.e. the nonrenormalized diagrams. The notation $(f)_1$ is used for the residue at the simple pole in Laurent expansion of a function f. The formula (2.10) gives the simple algorithm for the calculating of the critical indeces η and χ defined as

$$\gamma_\phi = \eta/2 \quad \text{and} \qquad \gamma_\psi = -\eta - \chi \tag{2.11}$$

up to $1/N^2$ order:

$$\eta = 2 \sum n_i (\mathcal{R}'\Gamma_{\phi\phi,i})_1, \qquad \chi = -2 \sum n_i (\mathcal{R}'\Gamma_{\phi\phi\psi,i})_1. \tag{2.12}$$

The advantages of the formula (2.10) for the practical calculations in the comparison with those (2.8) is self-evident. Our next purpose is to derive the analogous formulae in the case of arbitrary composite operator. However, to do this we shall take the advantage of the another approach.

3 The extended model

Let us consider the model based on the action obtained from (2.1) by introducing two new independent couplings u and v:

$$S = -\frac{1}{2}(\partial\phi^A)^2 - \frac{u}{2} M^{-2\Delta} \psi K_\Delta \psi + \frac{1}{2}\psi\phi^2 + \frac{v}{2}\psi K \psi. \tag{3.1}$$

In the following we will refer to it as UV model. It had been firstly introduced and investigated in Ref. [22]. Obviously, the initial model (2.1) is recovered by the special choice of parameters $u = v = 1$. As distinct from the ordinary sigma model the propagator of ψ field in the UV

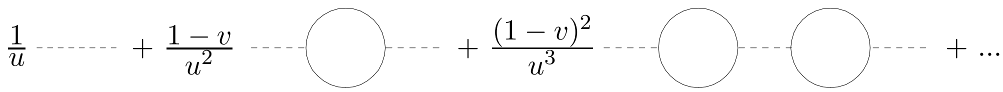


Figure 2: The effective ψ - line

model has the more complicated form. Indeed, at $v \neq 1$ the last term in the action (3.1) does not ensure the exact cancellation of the simple ϕ loop insertion in ψ line and one should sum up all such insertions (see Fig. 2), that yields for the propagator of ψ field:

$$G_\psi(p;u,v) \equiv \frac{1}{u} G_\psi(p) \left(1 + \frac{v-1}{u} t^\Delta + \frac{(v-1)^2}{u^2} t^{2\Delta} + \dots \right). \tag{3.2}$$

Here $t \equiv C(\Delta)(M^2/p^2)$. The theory is evidently renormalizable and the renormalized action takes form:

$$S_R = -\frac{Z_1}{2}(\partial\phi^A)^2 - \frac{u}{2} M^{-2\Delta} \psi K_\Delta \psi + \frac{Z_2}{2}\psi\phi^2 + \frac{v}{2}\psi K \psi, \tag{3.3}$$

where the renormalization constants depend on the coupling u, v also ($Z_i = Z_i(u,v,..)$). The redefinition of the fields and the couplings

$$\Phi_0 = Z_\Phi \Phi, \quad u_0 = u M^{-2\Delta} Z_u, \quad v_0 = v Z_v, \tag{3.4}$$

where $\Phi \equiv \{\phi, \psi\}$, $Z_\phi = Z_1^{1/2}$, $Z_\psi = Z_2 Z_1^{-1}$ and $Z_u = Z_v = Z_\psi^{-2}$, brings the action (3.3) into the initial form, hence, the UV model is the multiplicatively renormalizable. This implies that the model can be analyzed by the standard **RG** methods. The basic **RG** functions are defined as follows:

$$\gamma_\Phi = M\partial_M \ln Z_\Phi, \quad \beta_u = M\partial_M u = 2\Delta u - 2u\gamma_\psi, \quad \beta_v = M\partial_M v = -2v\gamma_\psi. \tag{3.5}$$

The **RG** equation for the one particle irreducible ($1PI$) Green functions reads as:

$$(M\partial_M + \beta_U \partial_U - n_\Phi \gamma_\Phi)\Gamma(p_1, \dots, p_n) = 0. \tag{3.6}$$

Here we used the shorthand notations $U = \{u, v\}$, $\beta_U \partial_U = \beta_u \partial_u + \beta_v \partial_v$ and $n_\Phi \gamma_\Phi = n_\phi \gamma_\phi + n_\psi \gamma_\psi$. For the set of the composite operators $\{F_i\}$ mixing under renormalization the corresponding equation has the matrix form:

$$\left([M\partial_M + \beta_U \partial_U - n_\Phi \gamma_\Phi]\, \delta^{ik} + \gamma_F^{ik} \right) \Gamma_k(p; p_1, \dots, p_n) = 0. \tag{3.7}$$

Here $\Gamma_i(p; p_1, \dots, p_n)$ is the n-points $1PI$ Green function with insertion of the renormalized operator $[F_i]_R$. The latter is defined as $[F_i]_R = Q_{ik} F_k$ and Q_{ik} is chosen to ensure finiteness of all Green functions of operator $[F_i]_R$. As usual, the summation over repeated indexes is implied.

At last the matrix of anomalous dimensions γ_F is defined as

$$\gamma_F = -M\partial_M Z_F Z_F^{-1}, \quad Z_F^{ik} = Q^{ik} Z_\Phi^{-n_{\Phi,k}}. \tag{3.8}$$

Here $Z_\Phi^{-n_{\Phi,k}} \equiv Z_\phi^{-n_{\phi,k}} Z_\psi^{-n_{\psi,k}}$ and the multiindex $n_{\Phi,k} \equiv \{n_{\phi,k}, n_{\psi,k}\}$ shows the number of the ϕ and ψ fields in the monomial F_k.

We remind, that our purpose is the calculation of the critical indices of composite operators in the ordinary sigma model, i.e. we are interested in the limit $u, v \to 1$ of the UV model. However, in this limit the beta functions β_u, β_v are not zero. (Indeed, $\beta_u = \beta_v = \gamma_\psi \neq 0$.) Thus, generally speaking the point $u = v = 1$ is not the critical point of UV model and the **RG** functions γ_Φ, γ_F

calculated at $u = v = 1$ are not coincide with the anomalous dimensions of the fields (operators) in the ordinary sigma model. However, it had been shown in Ref. [22] that at the particular choice of the renormalization scheme (the so called scheme of subtraction at fixed momenta, we refer to it as the scheme I) the renormalized Green functions depend on the difference $u - v$ only. In this case the anomalous term

$$\mathcal{A} \equiv -\beta_U \partial_U \Gamma_i^I(u - v; p, p_1, \dots, p_n)|_{u=v=1} \tag{3.9}$$

is dropped out of the Eq. (3.7) and the **RG** functions γ_Φ, γ_F calculated in this scheme are the true anomalous dimensions. The Green functions calculated in any other renormalization scheme are related to those $\Gamma_i^I(\bar{u} - \bar{v}, ...)$ calculated in the scheme I by a finite renormalization:

$$\Gamma_i(u, v, ...) = Z_{ik}^*(u, v) \cdot \Gamma_k^I(\bar{u} - \bar{v}, ...). \tag{3.10}$$

Here $\bar{u} = Z_u^*(u, v) \cdot u$, $\bar{v} = Z_v^*(u, v) \cdot v$ and $Z_{ik}^*(u, v)$, $Z_u^*(u, v)$, $Z_v^*(u, v)$ are the constants of the finite renormalization. Taking into account that the bare couplings u_0, v_0 are independent from the renormalization scheme, we obtain from the Eq. (3.4) that $u/v = \bar{u}/\bar{v}$ and, hence, $Z_u^* = Z_v^*$. Then, from the Eq. (3.10), one immediately obtains that Green functions calculated in the different regularization schemes at $u = v = 1$ differ one from another only by a constant multiplier. This proves that the critical exponents calculated in the model based on action (2.1) are independent from the regularization scheme being used and, in particularly, from the choice of the function $C(\Delta)$.

Further, inserting Γ_i in the form (3.10) into Eq. (3.7) one obtains the following equation for the Green functions of the ordinary sigma model:

$$\Big([M\partial_M - n_\Phi \gamma_\Phi] \, \delta^{ik} + (\gamma_F^{ik} + \gamma_F^{*,ik}) \Big) \, \Gamma_k(p; p_1, \dots, p_n) = 0. \tag{3.11}$$

where $\gamma_F^* = (\beta_u \partial_u + \beta_v \partial_v) Z^*(u, v) Z^{*-1}(u, v)|_{u=v=1}$. Thus the anomalous dimensions are given by the eigenvalues of the matrix $\gamma_F + \gamma_F^*$, but not of the γ_F alone. Of course, the γ_F^* is zero in the scheme I, but the latter is not very suitable for the practical calculations. The most convenient scheme for the calculations of **RG** functions is the **MS** scheme, but then one has to know γ_F^*. Fortunately, the matrix γ_F^* is appeared to be of $O(1/N^3)$ in the **MS** scheme, i.e. for the determination of the critical exponents up to $1/N^2$ order it is sufficient to know γ_F only. Henceforth we restrict ourselves to **MS** scheme only. The **RG** functions γ, then, are related to the simple Δ poles of the corresponding renormalization constants:

$$\gamma_\psi = 2\partial_u Z_\psi^{(1)} \Big|_{u=v=1}, \quad \gamma_\phi = 2\partial_u Z_\phi^{(1)} \Big|_{u=v=1}, \quad \gamma_F^{ik} = -2\partial_u Q_{ik}^{(1)} \Big|_{u=v=1} + \delta_{ik} n_{\Phi,k} \gamma_\Phi, \tag{3.12}$$

where $Z_i(u, v) = 1 + \sum_{n=1}^\infty Z_i^{(n)}(u, v)/\Delta^n$, $Q_{ik}(u, v) = \delta_{ik} + \sum_{n=1}^\infty Q_{ik}^{(n)}(u, v)/\Delta^n$. The mixing matrix Q_{ik} is calculated directly from the diagrams as the divergent part of the functional $\Gamma_{F_i}(x; \Phi)$:

$$\mathcal{K}\mathcal{R}' \Gamma_{F_i}(x; \Phi) = -\sum_{n=1}^\infty \frac{1}{\Delta^n} \cdot Q_{ik}^{(n)} F_k(x), \tag{3.13}$$

$$\Gamma_{F_i}(x; \Phi) \equiv \sum_n \frac{1}{n!} \int \mathrm{d}x_1 ... \mathrm{d}x_n \Gamma_{F_i, n}(x; x_1 ... x_n) \Phi(x_1) ... \Phi(x_n).$$

Here $\Gamma_{F_i}(x; x_1, \dots, x_n)$ is the nonrenormalized (i.e. one arising at averaging with action (3.1)) n-points $1PI$ Green function with operator F_i insertion and the operation $\mathcal{K}$ selects the singular (Δ poles) part of the diagrams.

Because there are not derivatives with respect v in the formulae (3.12), we can put $v = 1$ from the very beginning. In this case the propagator of ψ field (3.2) is reduced up to factor $1/u$ to that in the ordinary sigma model: $G_\psi(p, u, v = 1) = G_\psi(p)/u$. Then taking into account that the constants Z_i, Q_{ik} are determined by the divergent (pole) parts of the corresponding Green functions and that operation ∂_u gives simply the factor $-n_\psi$ (n_ψ is the number of ψ lines) for each diagram, one concludes that all calculations needed for determining of anomalous dimensions (3.12) can be done in the framework of the standard σ model.

Let us prove now that matrix γ^*_F vanishes in the $1/N^2$ order. For this it is sufficient to show that the anomalous term $\mathcal{A}$ (see Eq. (3.9)) is zero in this order. First of all, note, γ_ψ being of $O(1/N)$ order, we need to prove that the remaining part is of $O(1/N^2)$ order. Let us consider the arbitrary first order diagram of the ordinary σ model with n internal ψ lines. Regarding the regulator Δ on each line as the independent variable and picking out the multiplier $C(\Delta)$ from the ψ propagator we write the answer for the diagram as $C^n(\Delta)G(\Delta, \ldots, \Delta)$. Then, taking into account the form of the ψ propagator Eq. (3.2) one obtains the following expression for this diagram in the extended model:

$$\frac{1}{u^n} \sum_{m_1,\ldots,m_n=1}^{\infty} (C(\Delta))^{m_1+\ldots+m_n} G(m_1\Delta, \ldots, m_n\Delta)(\frac{v-1}{u})^{m_1+\ldots+m_n-n}. \tag{3.14}$$

As usual, the renormalized diagram is obtained from the nonrenormalized one by the $\mathcal{R}$ operation. In the Ref. [22] it was shown that the $\mathcal{R}$ operation based on the subtraction scheme I gives for each term in the sum (3.14) the same (up to factor $((v-1)/u)^{\cdots}$) answer. (Then, the sum (3.14) becomes a trivial and results in the factor $(1+u-v)^{-n}$.) The above property holds no longer in the **MS** scheme. The action of the $\mathcal{R}$ operation on different terms in the sum (3.14) leads to the essentially different answer even in the first order of $1/N$ expansion. However, in the first order of the $1/N$ expansion the whole sum (3.14) after renormalization depends on the difference $u-v$ only too.

In the case of a convergent diagram G this statement is trivial. Let us consider the superficially divergent diagram. In the $1/N$ order such diagram having no subdivergencies, we represent function $G(\ldots)$ as:

$$G(m_1\Delta, \ldots, m_n\Delta) = \frac{F(m_1\Delta, \ldots, m_n\Delta)}{(m_1 + \ldots + m_n)\Delta}, \tag{3.15}$$

where the function F is regular in the neighborhood of zero in all its arguments. Since in the **MS** scheme the R operation is reduced to the removing of the Δ poles one has for the renormalized diagram:

$$\frac{1}{u^n} \sum_{m_1,\ldots,m_n=1}^{\infty} \left(C'(0)F(0) + \frac{m_1F_1'(0) + \ldots + m_nF_n'(0)}{m_1 + \ldots + m_n} \right) (\frac{v-1}{u})^{m_1+\ldots+m_n-n}. \tag{3.16}$$

Taking into account that the terms like $m_1/(m_1 + \ldots + m_n)$ due to the evident symmetry of the rest of summand in m_i can be replaced by $1/n$, one obtains for the sum (3.16)

$$\frac{nC'(0)F(0) + F_1'(0) + \ldots + F_n'(0)}{n(1+u-v)^n} \tag{3.17}$$

The case when a superficially convergent diagram has a divergent subgraph can be treated along these lines as well.

Since the anomalous term $\mathcal{A}$ is proportional to $(\partial_u + \partial_v)\Gamma(p_i, u, v)|_{u=v=1}$ we conclude that $\mathcal{A}$, and hence, γ^* vanish up to $1/N^2$ order. (Note also, that we proved the more strong statement

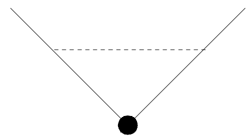


Figure 3: The $1/N$ - diagrams for the Green function with the insertion of the operator $\phi_A\phi_B$.

$A(u,v) = 0$ at $u = v$, than it was needed: $A(u,v) = 0$ at $u = v = 1$.) Unfortunately, the arguments used above do not work in the next orders of $1/N$ expansion, since the diagrams which give the nonzero contributions to the anomalous term $\mathcal{A}$ can be easy found in the higher orders. Though we cannot to reject the possibility of the cancellation of the contributions from the different diagrams, it seems to be very unlikely.

Thus we have shown that up to $1/N^2$ order the anomalous dimensions of composite operators are simply related to the Δ poles of the corresponding diagrams. The simplicity of the formulae (3.12) in the combination with the uniqueness of the triple vertex (at $\Delta = 0$) and masslessness of propagators makes the $1/N^2$ order calculations a rather straightforward task. The only thing which causes some problem is a large number of diagrams to be evaluated. In the next section we show how to overcome this difficulty.

4 Conformal structures in perturbation theory.

Everyone being a familiar with the $1/N$ expansion knows that the number of diagrams to be evaluated increases drastically with a order of expansion. To receive the impression about the rate of the growth it is sufficient to compare the number of the first and the second order diagram for any operator. Indeed, one has only one $1/N$ order diagram (Fig. 3) and eight ones in $1/N^2$ order (Fig. 4) for the $\phi^A\phi^B$ operator. The analogous comparison for the ψ^2 operator gives 3 and 69 diagrams, respectively. However, it can be easy realized that the bulk of the second order diagrams are the nothing else as the first order ones with the insertion of the vertex or the propagator corrections. It is intuitively clear that taking into account of these corrections must lead to the dressing of the bare propagators and vertices. In what follows we shall show that this really occurs. For further discussion to be more transparent we will use $\phi^A\phi^B$ operator as illustrative example. The second order diagrams for the two-point Green function with operator $\phi^A\phi^B$ insertion are drawn on Fig. 4. Two of them (c,d) arise due to the vertex corrections to the first order diagram (Fig. 3); one (e) – due to the self-energy (SE) insertion in ϕ line and three (f,g,h) – due to the SE insertions in ψ line.

We are interested in the contributions from the diagrams of this type (c – g on Fig. 4) to the matrix $\partial_u Q^{ik}|_{u=1}$. The more precisely we need the simple Δ pole term in the sum

$$\sum_{\{\Gamma_i\}} n_i \left(\mathcal{R}'\Gamma_i\right). \tag{4.1}$$

Here sum runs over the $1/N^2$ order nonrenormalized diagrams $\{\Gamma_i\}$ which arise from the $1/N$ order – base – diagrams after taken into account the vertex and the propagator corrections (the diagrams c-h on the Fig. 4). For later convenience we rewrite the sum (4.1) in the following form:

$$\sum_{\{\Gamma_i\}} n_i \left(\mathcal{R}'\Gamma_i\right) = \left(\sum_{\{\Gamma_i\}} n_i\Gamma_i - \sum_{\{\bar\Gamma_i\}} \bar n_i\bar\Gamma_i\right) - \sum_{\{\bar\Gamma_i\}} \widehat{n}_i\bar\Gamma_i. \tag{4.2}$$

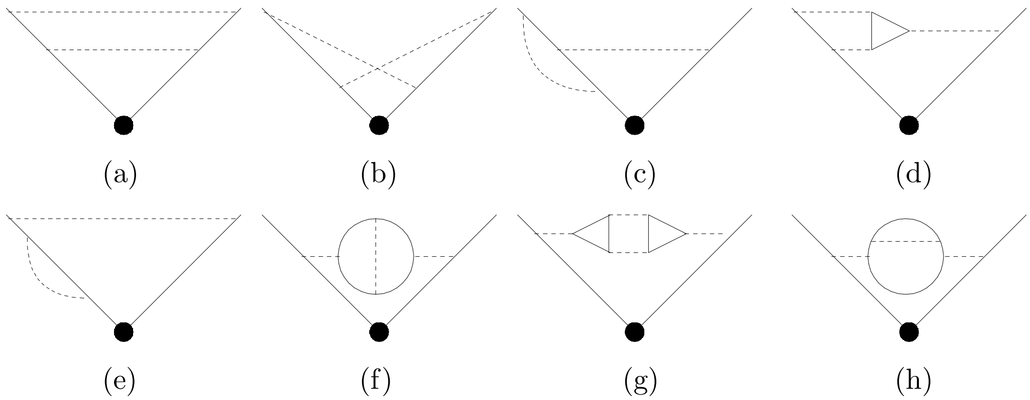
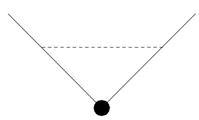
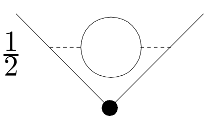


Figure 4: The $1/N^2$ - diagrams for the Green function with insertion of operator $\phi_A \phi_B$.

Here $\{\bar{\Gamma}_i\}$ is the set of the counterterm diagrams corresponding to the ones from the set $\{\Gamma_i\}$, i.e $\mathcal{R}'\Gamma_i = \Gamma_i - \bar{\Gamma}_i$; $\bar{n}_i$ is the number of ψ lines in the diagram $\bar{\Gamma}_i$, while $\hat{n}_i = n_i - \bar{n}_i$ is the number of ψ lines in the divergent subgraph. It is appeared that the last sum in the Eq. (4.2) has no simple Δ pole, provided that the relation $2n_\psi = n_\phi = n_V$ between the number of the internal lines and the triple vertices holds for the $1/N$ order base diagram. Indeed, one can see that the divergent subgraphs in the diagrams with the SE insertions in ψ line (Fig. 4 f–h) are the same as those in the diagrams with the vertex corrections and the SE insertion in ϕ line (Fig. 4 c–e). We combine the diagrams into three different groups — (c,f), (d,g), (e,h) – in accordance with the type of the divergent subgraph. Obviously, the diagrams appearing after reducing of the divergent subgraph in the diagrams from the same group, i.e. those from the set $\{\bar{\Gamma}_i\}$, differ one from another only by the simple ϕ loop insertion in ψ line. For example, for S/Δ being the pole part of the divergent subgraph, the contribution of the (e,h) pair into the sum in question reads:

$\frac{S}{\Delta} \times \Bigg($  $+\ \frac{1}{2}$  $\Bigg)$

(The factor 1/2 here is due to the symmetry coefficient of diagram (h).) The analytical expression for the above sum can be represented as

$$\frac{S}{\Delta} C(\Delta) \left(\frac{F(\Delta)}{\Delta} - C(\Delta) \frac{F(2\Delta)}{2\Delta} \right), \tag{4.3}$$

F being the regular function of Δ. (To derive (4.3) it is sufficient to remember the definitions (2.2),(2.3),(2.4)). It is seen the difference (4.3) does not contain the simple Δ pole. Obviously, the same arguments work and in a general case, the condition $2n_\psi = n_\phi = n_V$ ensuring the cancellation of the simple pole contributions among each group separately. Henceforth we shall only deal with the diagrams satisfying this condition. Moreover, we confine our consideration by those diagrams for which the only subdivergencies are due to the vertex and SE subgraphs. In spite of that these conditions restrict the class of the admissible operators it appears to be wide enough to justify the necessity of the present discussion.

We start with the analyzing of the diagrams with the SE insertions, say, in ϕ lines (Fig. 4 e). For later convenience we will regard the line indices as the variables and distinguish the regulators Δ on the ψ - lines being internal (Δ_1) and external (Δ_2) with respect to the divergent subgraph. Also, we introduce the regular function F related to the base diagram (Fig. 3) with the line

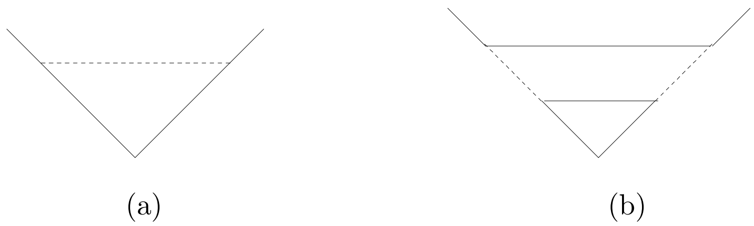


Figure 5: The $1/N$ order vertex diagrams

indeces differing from their canonical values $\alpha_\phi = 1$ and $\alpha_\psi = \mu - 2$ by Δ_i as follows

$$G = \frac{F(\alpha_\phi + \Delta_1, \alpha_\phi + \Delta_2, \alpha_\psi + \Delta_3)}{\Delta_1 + \Delta_2 + \Delta_3}. \tag{4.4}$$

Then the answer for the diagram with the SE insertion in ϕ line can be written as

$$\frac{S(\Delta_1)}{\Delta_1} \cdot \frac{F(\alpha_\phi + \Delta_1, \alpha_\phi, \alpha_\psi + \Delta_2)}{\Delta_1 + \Delta_2}, \tag{4.5}$$

where the factor $S(\Delta_1)/\Delta_1$ result from the integration over loop momenta in the subgraph. For the corresponding counterterm diagram one has $S(0)/\Delta_1 \cdot F(\alpha_\phi, \alpha_\phi, \alpha_\psi + \Delta_2)/\Delta_2$. Their contribution to the sum (4.2) reads ($n = 2$ and $\bar{n} = 1$ in this case)

$$2 \cdot \frac{S(\Delta_1)}{\Delta_1} \cdot \frac{F(\alpha_\phi + \Delta_1, \alpha_\phi, \alpha_\psi + \Delta_2)}{\Delta_1 + \Delta_2} - \frac{S(0)}{\Delta_1} \cdot \frac{F(\alpha_\phi, \alpha_\phi, \alpha_\psi + \Delta_2)}{\Delta_2}. \tag{4.6}$$

Evaluating of the above difference at the "symmetric" point $\Delta_1 = \Delta_2 = \Delta$ yields for the simple pole residue

$$S(0)F_1'(\alpha_\phi, \alpha_\phi, \alpha_\psi) + S'(0)F(\alpha_\phi, \alpha_\phi, \alpha_\psi). \tag{4.7}$$

Noticing that the constant $-S(0)$ is nothing else as the anomalous dimension of ϕ field: $\gamma_\phi = -S(0) + O(1/N^2)$ we relate the above expression to the base diagram with one bare ϕ propagator being replaced by the dressed one: $1/p^2 \to A/p^{2-\eta}$ ($A = 1 + S'(0)$ and $\eta = 2\gamma_\phi$). Namely, the expression (4.7) can be obtained as the $1/N^2$ - order term in the expansion of the following object

$$A \cdot F(\alpha_\phi - \gamma_\phi, \alpha_\phi, \alpha_\psi). \tag{4.8}$$

It is not more hard to see that taking into account the SE insertions in ψ line also leads to the dressing of this line, i.e. the contributions of these diagrams to the sum (4.2) can be received from the expansion of the object:

$$B \cdot F(\alpha_\phi, \alpha_\phi, \alpha_\psi - \gamma_\psi). \tag{4.9}$$

The following necessary step is the inclusion into consideration of the vertex corrections. Indeed, having restricted by the propagators corrections only one loses the more attractive feature of $1/N$ - expansion — the uniqueness of the triple vertex. The analysis of the vertex corrections is a bit more complicated than those of the SE insertions. Indeed, the diagram with insertion in a line has, in fact, the same topology as the initial one, and hence, (that is crucial) they both can be described by one and the same function. To achieve this in the case in question, where the topology of the diagrams is essentially different, we proceed in the following way. First of all, let us consider the vertex function itself. Up to terms of $O(1/N^2)$ order it reads

$$\gamma_R(p, q) = 1 + \frac{\gamma_1(\Delta; p, q)}{\Delta} + \frac{\gamma_2(\Delta; p, q)}{2\Delta} - \text{poles} = 1 + \gamma_1'(p, q) + \frac{1}{2}\gamma_2'(p, q). \tag{4.10}$$

Here $\gamma_1(\Delta)/\Delta$, $\gamma_2(\Delta)/2\Delta$ are the contributions of the diagrams shown on the Fig. 5a,b, respectively. (In the further we shall not show explicitly the momentum dependence in the functions γ_i, γ_R. Note only, that at $\Delta = 0$ the functions γ_i are independent from p, q.) Due to conformal invariance of the theory, the form of the *full* vertex γ_R is fixed up to the constant factor. In coordinate space it reads

$$\gamma_R(x, y; z) \equiv \hat{Z}(z-x)^{-2\alpha}(z-y)^{-2\alpha}(x-y)^{-2\beta}, \tag{4.11}$$

where $\alpha = \mu - 1 - \gamma_\psi/2$, $\beta = 2 - \gamma_\phi + \gamma_\psi/2$.

Further, taking in mind the pole structure of the diagrams (c,d) on Fig. 4 we write for the latter

$$\frac{\widehat{F}[\gamma_m(\Delta_1); \Delta_2]}{m\Delta_1(m\Delta_1 + \Delta_2)}, \tag{4.12}$$

where $m = 1, 2$, respectively. The function $\widehat{F}$ is regular with respect the variables $\Delta_{1(2)}$ and is defined as follows :

$$\widehat{F}[\gamma_m(\Delta_1); \Delta_2] = (m\Delta_1 + \Delta_2) \int \frac{d^{2\mu}p}{(2\pi)^{2\mu}} \gamma_m(\Delta_1, p, p+q)(G/\gamma_1)(p, ..., \Delta_2). \tag{4.13}$$

Here the function $(G/\gamma_1)(p, ...\Delta_2)$ is related to the diagram with the vertex subgraph removed. (For the diagram on Fig. 4c,d it is given by the product of three propagators). The evaluation of the contribution of diagrams (c,d) on Fig. 4 together with the corresponding counterterm diagrams yields for the simple pole residue in the sum (4.2)

$$\partial_\Delta \left(\widehat{F}[\gamma_1(\Delta); 0] + \frac{1}{2}\widehat{F}[\gamma_2(\Delta); 0] \right) \Big|_{\Delta=0}. \tag{4.14}$$

Looking on the Eq. (4.13) one can see that the renormalized vertex γ_R should appear in the natural way if it were be possible to carry out differentiating with respect Δ under the sign of integral. However, at $\Delta_2 = 0$ the integral evolves the pole in Δ_1, therefore this operation is not allowed. To obviate this difficulty, one can, taking into account the regularity of the function $\widehat{F}$ in $\Delta_{1(2)}$, carry out differentiation at the nonzero second argument Δ_2 and then take the limit $\Delta_2 \to 0$. At first step we obtain for (4.14)

$$\frac{\widehat{F}(\gamma_1(0) + \gamma_2(0), \Delta_2)}{\Delta_2} + \Delta_2 \int \frac{d^{2\mu}p}{(2\pi)^{2\mu}} \gamma_R^{(1)}(p, p+q)(G/\gamma_1)(p, ..., \Delta_2), \tag{4.15}$$

where $\gamma_R^{(1)} = \gamma_1' + \gamma_2'/2$ (see Eq. (4.10)). To proceed further we represent $\gamma_R^{(1)}$ as $\gamma_R^{(1)} = \partial_t \gamma_R(t/N)|_{t=0}$ and, again, change the order of the differentiation and integration. At last, taking into account that scaling dimension χ (2.12) of the function γ_R is equal to $-2(\gamma_1(0) + \gamma_2(0))$ at first $1/N$ order, so that the integral in (4.15) evolves the pole in $(\Delta_2 - t\chi/2)$, we find eventually for (4.14) the following representation

$$\partial_t \left(-\frac{t\chi}{2} \int \frac{d^{2\mu}p}{(2\pi)^{2\mu}} \gamma_R(t/N, p, p+q)(G/\gamma_1)(p, ..., 0) \right) \Big|_{t=0}. \tag{4.16}$$

In other words the contribution of the diagrams in question is obtained from the $1/N$ expansion of the diagram with the dressed vertex.

Basing on these results (Eqs. (4.8),(4.9),(4.16)) we can formulate the simple algorithm for the calculating of the contribution of the diagrams with the propagator and vertex corrections to the matrix of the anomalous dimensions. Let G to be a $1/N$ order diagram and the diagrams $\{G_i\}$ are the $1/N^2$ order ones from the class under consideration. Then to obtain the contribution of the diagrams $\{G_i\}$ to the anomalous dimension one need:

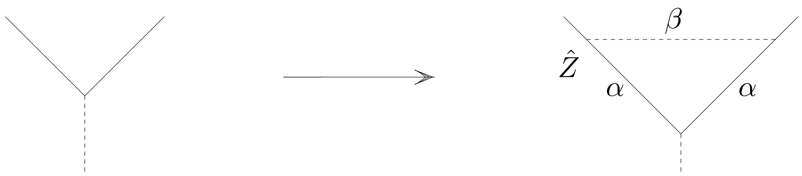


Figure 6: The conformal triangle.

1. replace in the base diagram G the bare propagators by dressed ones and the pointlike triple vertex by the conformal triangle (see Fig. 6):

$$\hat{G}_\phi(x) \equiv \frac{\hat{A}}{x^{2\Delta_\phi}}, \qquad \hat{G}_\psi(x) \equiv \frac{\hat{B}}{x^{2\Delta_\psi}}, \tag{4.17}$$

$$\gamma_R(x,y;z) \equiv \hat{Z}(z-x)^{-2\alpha}(z-y)^{-2\alpha}(x-y)^{-2\beta}. \tag{4.18}$$

   Here $\alpha = \mu - 1 - \gamma_\psi/2$, $\beta = 2 - \gamma_\phi + \gamma_\psi/2$ and Δ_ϕ, Δ_ψ are the full scaling dimensions of the fields ϕ, ψ, respectively. $\Delta_\phi = \mu - 1 + \gamma_\phi$, $\Delta_\psi = 2 + \gamma_\psi$.

2. introduce a regulator Δ in anyone line (It is needed because after the above changes one obtains a superficially divergent diagram).

3. evaluate the Δ pole residue in the resulting diagram and pick out the $1/N^2$ order term – $G^{(2)}$. Then the required contribution is given by $-2G^{(2)}$.

The explicit expressions for the amplitudes $\hat{A}$, $\hat{B}$, $\hat{Z}$ are given in Appendix A.

Following these prescriptions one need calculate only one –"conformal" diagram (see Fig. 7) – instead of the six $1/N^2$ order diagrams shown on Fig. 4 (c-h). It should be noted also, that the resulting diagram, unlike the original ones, has superficial divergence only, that, in its turn, gives the definite advantages at the calculations. We end this section with remark, that the above procedure being useful even if the condition $2n_\psi = n_\phi = n_V$ is not fulfilled. Indeed, the only changes which will occur are that the last sum in the Eq. (4.2) will not be zero any longer and will contribute to the anomalous dimension on the equal foot with the first ones.

5 The anomalous dimensions of the $(\otimes\vec{\Phi})^s$ and $(\vec{\Phi}\otimes(\otimes\vec{\partial})^n\vec{\Phi})$ operators in $1/N^2$ order

In this section we apply the technique developed above to calculate the anomalous dimensions of the operators

$$F_1^s \;=\; (\otimes\vec{\Phi})^s \equiv \mathrm{Sym}\phi^{A_1}\ldots\phi^{A_s} \;-\; \text{traces}, \tag{5.19}$$

$$F_2^n \;=\; (\vec{\Phi}\otimes(\otimes\vec{\partial})^n\vec{\Phi}) \equiv \mathrm{Sym}\phi^{A}\partial_{i_1}\ldots\partial_{i_n}\phi^{B} \;-\; \text{traces} \tag{5.20}$$

in the $1/N^2$ order. Here, the symbol Sym implies the symmetrization over the internal indeces as well as the spatial ones.

The operators F_1 are multiplicatively renormalized, whereas F_2 does not. However, the operators admixing to F_2 being the total derivatives, they are irrelevant for our purposes. The diagrams to be evaluated for the determination of the anomalous dimensions of operators F_1, F_2 are those shown on Fig. 4 and Fig. 8, the latter being obviously relevant only for the F_1 operators. Because the diagrams $c-h$ on Fig. 4 can be replaced by one "conformal" diagram (Fig 7) in

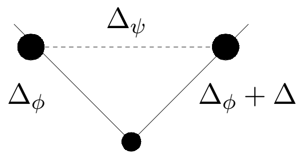


Figure 7: The "conformal" diagram. The black circles denote the conformal triangle.

accordance with the rules formulated in the previous section, one has to calculate 5 diagrams for the F_1 operators and 3 ones for the F_2 operators.

We do not dwell upon the technique of the calculating of the diagrams, since it is discussed in very details in the papers [1, 2, 4, 23, 24] and give the final results for the anomalous dimensions only. (For completeness, we give the explicit expressions for the pole terms of each diagram in Appendix B and the some useful formulae are collected in Appendix C.)

The anomalous dimensions $\gamma_{(s)}$ and $\gamma_{(n)}$ of the operators F_1^s and F_2^n it is convenient to represent as

$$\gamma_{(s)} = u_{(s)} + s\gamma_\phi \qquad \gamma_{(n)} = u_{(n)} + 2\gamma_\phi, \tag{5.21}$$

where γ_ϕ is the anomalous dimension of the field ϕ known up to $1/N^3$ order [4]. Then representing u in the following form $u = \sum_{k=1}^{\infty} u^{(k)}/N^k$ we have for the two first terms:

$$u^1_{(s)} = -\frac{s(s-1)\mu}{2(\mu-2)}\eta_1, \tag{5.22}$$

$$\begin{aligned} u^2_{(s)} &= -\eta_1^2 \frac{s(s-1)\mu}{4(\mu-1)(\mu-2)^2}\Big\{2(s-2)\mu(\psi'(1)-\psi'(\mu)) \\ &\quad +[\mu(2\mu-3)+2(\mu-1)(2\mu^2-3\mu+2)R]\Big\}, \end{aligned} \tag{5.23}$$

$$u^1_{(n)} = -\frac{\mu(\mu-1)}{(\mu-1+n)(\mu-2+n)}\eta_1, \tag{5.24}$$

$$\begin{aligned} u^2_{(n)} &= -\eta_1^2 \frac{\mu(\mu-1)}{(\mu-1+n)(\mu-2+n)}\Big\{\frac{(2\mu^2-3\mu+2)}{\mu-2}R - \frac{2(\mu-1)(2\mu-1)}{\mu-2}S(n,\mu) \\ &\quad +\frac{1}{2}\mu(\mu-1)R(n,\mu) + \frac{1}{\mu-2}\left(\frac{n}{\mu-2+n} + \frac{\mu(\mu^2-5\mu+5)}{(\mu-1)(\mu-2)}\right) \\ &\quad -\frac{\mu(\mu-1)}{2(\mu-1+n)(\mu-2+n)}\left(1-\frac{1}{(\mu-1+n)}-\frac{1}{(\mu-2+n)}\right)\Big\}, \end{aligned} \tag{5.25}$$

where

$$\eta_1 = 4(2-\mu)\Gamma(2\mu-2)/\Gamma^2(\mu-1)\Gamma(2-\mu)\Gamma(\mu+1), \tag{5.26}$$

$$R = \psi(1) + \psi(\mu-1) - \psi(2-\mu) - \psi(2\mu-2), \tag{5.27}$$

$$S(n,\mu) = \psi(\mu-1+n) - \psi(\mu-1), \tag{5.28}$$

$$R(n,\mu) = \int_0^1\int_0^1 \mathrm{d}\alpha\mathrm{d}\beta\alpha^{\mu-3}\beta^{\mu-3}(1-\alpha-\beta)^n. \tag{5.29}$$

The critical exponents of the operators F_1, F_2 are known with the high accuracy in the $4-\epsilon$ as well as in the $2+\epsilon$ expansions. Expanding our result for $\gamma_{(s)}$ in the ϵ series near $d=2$ we have obtained the full agreement with the four – loop result of Wegner [25]. The same exponents are known up to ϵ^3 order in the $4-\epsilon$ expansion [26]. In this case we find out the discrepancy of

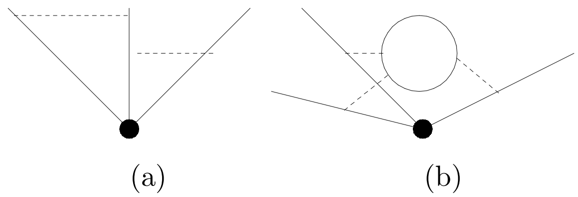


Figure 8: The $1/N^2$ order diagrams with insertion of operator F_1.

our result with those of Wallace and Zia [26] in term $\sim \epsilon^3 s(s-1)/N^2$ (in our answer this term enters with the coefficient (-10), against the coefficient (-8) in the paper [26]).

Further, using the results of the paper [27], where the four – loop anomalous dimensions of $\phi(\otimes\vec{\partial})^n\phi$ operators had been calculated in the scalar ϕ^4 model in the $4-\epsilon$ expansion, we reconstructed the answer for the $O(N)$ vector model by restoring the $O(N)$ coefficients for all diagrams. Comparing the expressions for the critical exponents of F_2 operators derived in the both ($1/N$ and $4-\epsilon$) approaches we found that they are in the full agreement up to ϵ^4/N^2 terms.

6 Conclusion

In the present paper we have developed the simple method for the calculating of the anomalous dimensions (up to $1/N^2$ order) in the $1/N$ expansion. In the comparison with the other well known methods of the conformal bootstrap and the selfconsistency equations this one has the advantage of being not restricted to the special class of the operators. In this approach the formulae relating the anomalous dimensions with the divergencies of the corresponding Feynman diagrams are very simple and in the one to one correspondence with their counterparts in the dimensional regularization scheme. These features together with the uniqueness of the triple vertex of the nonlinear sigma model make the $1/N^2$ order calculations of the rather straightforward task. In some cases (see for example the sec. 5) the calculations are not much more complicated than the two – loop ones in the $O(N)$ model in the $4-\epsilon$ expansion. However, in a general situation, beyond $1/N$ order, the number of the diagrams to be evaluated is usually large enough. We have shown that this problem is partially removed after going on to the diagrams with the dressed propagators and vertices. The number of the diagrams is considerably reduced by this trick and only slightly exceeds the number of those in the conformal bootstrap method, which is considered to be the most economic in this sense.

We conclude with remark, that though so far we consider the sigma model only, it is evident that the same technique can be apply to the many other models admitting $1/N$ expansion — $CP(N)$, Gross – Neveu and Thirring models, etc.

Acknowledgments

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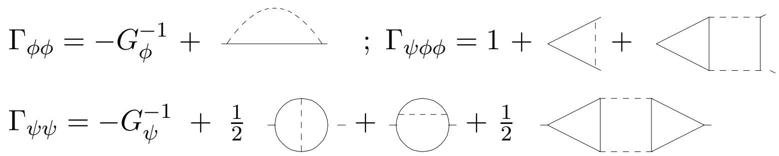

Figure 9: The $1/N$ order diagrams for the Green functions $\Gamma_{\phi\phi}$, $\Gamma_{\psi\psi}$ and $\Gamma_{\psi\phi\phi}$.

Appendices

A The renormalized Green functions in $1/N$ order

In this appendix we give the explicit expressions for the amplitudes $\widehat{A}$, $\widehat{B}$, $\widehat{Z}$ introduced into consideration in the Sec. 4. Obviously, these quantities depend on the our choice of the function $C(\Delta)$ (see Eqs. (2.3),(2.4)). However, it is easy to understand, that in the first $1/N$ order this dependence is almost trivial and can be taking into account by the redefinition of the parameter M. We specify $C(\Delta)$ by the condition for the propagator of ψ field in coordinate space (Eq.(2.4)) to have the amplitude independent from Δ, except from the factor $M^{2\Delta}$. Then one has for the amplitude A, B of the bare propagators in the coordinate space:

$$A = \frac{H(1)}{4\pi^\mu}, \qquad B = -M^{2\Delta}\frac{32H(2-\mu)}{H(2)H^2(1)}. \tag{A.1}$$

Evaluating the $1/N$ order diagrams for $1PI$ Green functions $\Gamma_{\phi\phi}$, $\Gamma_{\psi\psi}$ $\Gamma_{\psi\phi\phi}$ shown on Fig. 9 we obtain for the amplitudes $\widehat{A}$,$\widehat{B}$,$\widehat{Z}$:

$$\begin{aligned}
\widehat{A} &= AM^{-2\gamma_\phi}\cdot\left[1-\frac{\eta_1}{2N}\frac{\mu^2+\mu-1}{\mu(\mu-1)}\right], && \text{(A.2)}\\
\widehat{B} &= BM^{-2\gamma_\psi}\left[1+\frac{\eta_1}{N}\left(\frac{2\mu^2-3\mu+2}{\mu-2}\cdot R-\frac{4\mu^5-19\mu^4+25\mu^3-\mu^2-14\mu+4}{\mu(\mu-1)(\mu-2)^2}\right)\right], && \text{(A.3)}\\
\widehat{Z} &= -M^{-2\chi}\frac{\chi}{2}\frac{H^2(1)H(\mu-2)\Gamma(\mu)}{\pi^{2\mu}}\cdot\left[1+\frac{\eta_1}{N}\frac{(\mu-3)(6\mu^2-9\mu+2)}{(\mu-2)^2}\right]. && \text{(A.4)}
\end{aligned}$$

Here η_1, R are given by Eq. (5.26),(5.27) and γ_ϕ, γ_ψ are the known anomalous dimensions of the fields ϕ, ψ

$$\begin{aligned}
\gamma_\phi &= \frac{\eta_1}{2N}+O(1/N^2),\\
\gamma_\psi &= -\frac{2\eta_1}{N}\frac{(\mu-1)(2\mu-1)}{2-\mu}+O(1/N^2). && \text{(A.5)}
\end{aligned}$$

The index χ is defined as $\chi = -(\gamma_\psi + 2\gamma_\phi)$.

B Values for the $1/N^2$ order integrals

In this appendix we give the explicit expressions for the contributions to the anomalous dimensions $\gamma_{(s)}, \gamma_{(n)}$ of the corresponding Feynman integrals. We will use the labels A, B, C, D, E for the diagrams on the Fig. 4 a,b, Fig. 7 and Fig. 8 a,b, resp., as well as for the values of the

latters. In the case of the $F_1^{(s)}$ operators we have obtained for the simple Δ pole residue of the corresponding integrals in the units η_1^2/N^2:

$$A = -\frac{s(s-1)}{2}\frac{\mu^2(\mu^2-5\mu+5)}{8(\mu-1)(\mu-2)^3}, \tag{B.1}$$

$$B = \frac{s(s-1)}{2}\frac{\mu^2(\mu-1)}{8(\mu-2)^3}, \tag{B.2}$$

$$C = s(s-1)\frac{\mu}{(2\mu-2)^2}\left[\frac{\mu(\mu^2-5\mu+5)}{(\mu-1)(\mu-2)} + (2\mu^2-3\mu+2)R\right], \tag{B.3}$$

$$D = s(s-1)(s-2)\frac{\mu^2}{8(\mu-1)(\mu-2)^2}, \tag{B.4}$$

$$E = \frac{s(s-1)(s-2)}{6}\frac{3\mu^2(\mu-1)}{2(\mu-2)^2}C. \tag{B.5}$$

For diagrams with the insertion of the $F_2^{(n)}$ operators we derived

$$A = -\frac{\mu^2(\mu-1)^2}{8(\mu-1+n)^2(\mu-2+n)^2}\left(1-\frac{1}{\mu-1+n}-\frac{1}{\mu-2+n}\right), \tag{B.6}$$

$$B = \frac{\mu^2(\mu-1)^2}{8(\mu-1+n)(\mu-2+n)}R(n;\mu), \tag{B.7}$$

$$\begin{aligned} C = {} & \frac{\mu(\mu-1)}{2(\mu-1+n)(\mu-2+n)}\Big\{\frac{(2\mu^2-3\mu+2)}{\mu-2}R - \\ & -\frac{2(\mu-1)(2\mu-1)}{\mu-2}S(n,\mu) + \frac{1}{\mu-2}\left(\frac{n}{\mu-2+n}+\frac{\mu(\mu^2-5\mu+5)}{(\mu-1)(\mu-2)}\right)\Big\}. \end{aligned} \tag{B.8}$$

Each of these quantities enters into expression for the anomalous dimension with the coefficient $(-2n_\psi)$, if we accept $n_\psi = 1$ for diagram C.

C Chain rule and uniqueness relation

Here we collect some rules (see Refs. [2, 4, 23]) which form the basis for the calculations of diagrams in the massless theories. As usual, we will denote the propagator $1/|x-y|^{2\alpha}$ by the line with the index α.

The Fourier-transform of $|x|^{-2\alpha}$ reads

$$\int d^{2\mu}x\frac{e^{\imath px}}{x^{2\alpha}} = \pi^\mu\frac{2^{2\alpha'}H(\alpha)}{p^{2\alpha'}}. \tag{C.9}$$

We remind that $\mu \equiv D/2$, $H(z) \equiv \Gamma(\mu-z)/\Gamma(z)$ and $z' \equiv \mu - z$.

Further, we consider the chain of propagators, which is illustrated on Fig. 10. The point where the ends of propagators are joined is the vertex of integration. The result of this integration is a new line with index $\alpha+\beta-\mu$ and amplitude proportional to $\pi^\mu H(\alpha,\beta,2\mu-\alpha-\beta)$, where $H(\alpha_1,\alpha_2,\alpha_3) = \prod_{i=1}^3 H(\alpha_i)$.

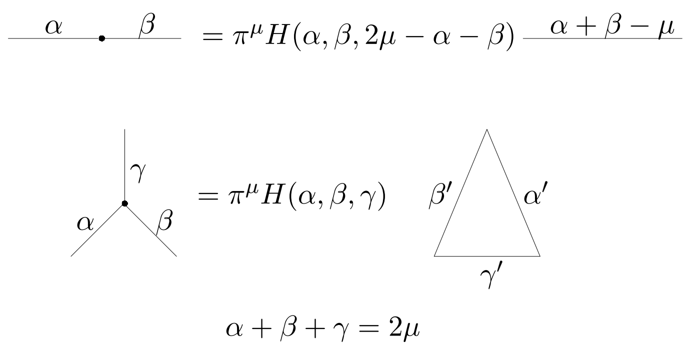

Figure 10: Chain rule and uniqueness relation

The other useful relation is the so-called uniqueness relation. In the case if the sum of the exponents of the lines meeting at the vertex is equal to the dimension of space-time it can be related to a triangle graph (see Fig. 10).

The corresponding generalization of these rules needed for the evaluation of the the diagrams with the insertion of the tensor operators, are given in Refs. [24]. We give here only that which is relevant for the treatment of the traceless and symmetric operators, like operator F_2 in our case. It is convenient to swamp the the spatial indices $\nu_1, ..., \nu_n$ of a operator by contracting with a constant vector u^i, such that $u^2 = 0$, and consider scalar operator $T_n(x, u) = T_n^{\nu_1 \cdots \nu_n}(x) u_{\nu_1} \ldots u_{\nu_n}$. In the course of calculations one should often to integrate the chains of the propagators like $(ux)^k / |x|^{2\alpha}$. Then, the corresponding rule for the integration of the chains reads

$$\int \mathrm{d}^{2\mu} x \frac{(x, u)^n (z - x, u)^m}{x^{2\alpha} (x - z)^{2\beta}} = \pi^{mu} \frac{H_n(\alpha) H_m(\beta)}{H_{n+m}(\alpha + \beta - \mu)} \frac{(u, z)^{m+n}}{z^{2(\alpha + \beta - \mu)}}, \tag{C.10}$$

where $H_n(z) \equiv \Gamma(\mu + n - z)/\Gamma(z)$.

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